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Quantum Congestion Game for Overcrowding Prevention within Airport Common Areas

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Abstract: Quantum game theory merges principles from quantum mechanics with game theory, exploring how quantum phenomena such as superposition and entanglement can influence strategic decision making. It offers a novel approach to analyzing and optimizing complex systems where traditional game theory may fall short. Congestion of passengers, if considered as a network, may fall into the categories of optimization cases of quantum games. This paper explores the application of quantum potential games to minimize congestion in common areas at airports. The players/passengers of the airport have identical interests and they share the same utility function. A metric is introduced that considers a passenger's visit to a common area by setting their preferences, in order to avoid congestion. Passengers can decide whether to visit a specific common area or choose an alternative. This study demonstrates that the proposed game is a quantum potential game for tackling congestion, with identical interests, ensuring the existence of a Nash equilibrium. We consider passengers to be players that want to ensure their interests. Quantum entanglement is utilized to validate the concept, and the results highlight the effectiveness of this approach. The objective is to ensure that not all passengers select the same common place of the airport to reduce getting crowded; hence, the airborne disease infection probability increases due to overcrowding. Our findings provide a promising framework for optimizing passenger flow and reducing congestion in airport common areas through quantum game theory. We showed that the proposed system is stable by encapsulating the Lyapunov stability. We compared it to a simulated annealing approach to show the efficacy of the quantum game approach. We acknowledge that this framework can be utilized in other disciplines as well. For our future work, we will research different strategies than binary ones to investigate the efficacy of the approach.

Keywords: airport; common place; quantum game; Lyapunov stability; identical interest; congestion; entanglement; occupancy; queue

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1. Introduction

The rapid spread of COVID-19 and other airborne illnesses presents significant health hazards, including fatalities. With its high transmission rate, COVID-19 poses a particularly grave risk in densely populated indoor environments, like airports, where people gather at various points such as duty-free shops, restaurants, and boarding gates. Consequently, there is an urgent need to regulate passenger movement within airport facilities to alleviate overcrowding. Recent research has focused on modeling the indoor spread of COVID-19 and analyzing human behavior in such environments, offering valuable insights into effective strategies for preventing transmission [1,2]. The primary modes of transmission are through respiratory droplets and direct contact between individuals. Implementing measures to control the density of individuals per square meter in airport spaces can effectively mitigate the risk of virus transmission. Additionally, tracking passengers from their arrival or check-in, and throughout their airport journey, can play a crucial role in managing passenger flow and minimizing transmission risks. A person moving to airport common areas can be regarded as a congestion avoidance situation, where persons are players that aim to minimize their exposure to overcrowding in airport common areas.

Network congestion games, also known as atomic selfish routing games in the academic literature [3], are characterized by a directed graph, several pairs of source-target vertices, and non-decreasing cost functions assigned to each edge in the graph. For each pair of source and target vertices, a player is tasked with selecting a route from the source to the target vertex. The cost incurred by a player is determined by the number of other players who opt for paths that share edges with their chosen path, as well as the cost functions associated with these edges. In this framework, a Nash equilibrium assigns each player to a specific path, ensuring that no player has an incentive to deviate, since changing their path would not result in a lower cost [4].

It has been proven that network congestion games adhere to the framework of potential games, ensuring the existence of Nash equilibria. Monderer and Shapley [5] investigated potential games, providing detailed insights into iterative methodologies employing best-response strategies to converge towards equilibria. For a comprehensive introduction and foundational understanding of general routing games, readers are encouraged to consult [6].

Games can fall into two categories, classical or quantum, depending on whether they utilize classical or quantum resources/strategies. Quantum games present several advantages over classical ones, including enhanced winning probabilities, efficiency, payoffs, and equilibria [7]. Moreover, quantum strategies yield consistently higher average payoffs compared to classical strategies in competitive scenarios where involved entities have conflicting interests.

Traditional games are frequently limited by classical probability models and computational constraints, which can impede optimal decision making, particularly in highly interconnected or complex, dynamic systems. Quantum games overcome these limitations by utilizing principles of quantum mechanics, such as superposition and entanglement, to broaden strategic options and increase efficiency in outcomes. By offering more extensive strategy spaces and introducing new equilibrium concepts, quantum games can enhance strategic effectiveness and solution precision in ways that classical game models are unable to match.

Quantum games, by integrating entanglement from the outset, facilitate multiple players in achieving satisfactory outcomes through diverse strategies. Unlike classical games, which are reliant on deterministic tactics favoring singular maximum payoffs, quantum strategies embrace convex linear combinations of unitary actions, enabling multiple players to optimize expected returns [8]. By formulating quantum strategies within a convex compact subset of a finite-dimensional vector space, game equilibria are ensured, as per Glicksberg's extension of the Nash equilibrium [9]. There exist some good surveys in the literature regarding quantum games that start from the quantum mechanics side of the research domain [10–12].

Players limiting themselves to only classical strategies risk consistently losing to a player using quantum strategies [7]. Thus, rationality suggests that all players in a quantum game would adopt quantum strategies to maintain a fair competition. Also, Prisoner's dilemma is "quantumized", allowing both players access to quantum strategies. Leveraging these strategies to accomplish a better payoff than in the classical version needs examination [13]. Moreover, probability plays a fundamental role in quantum mechanics. Given that classical games also rely on probability, the intersection between classical and quantum game theory offers a conceptually rich framework. Furthermore, since quantum mechanics underpins the laws of nature, its principles are likely reflected in human thought processes and communication. Exploring quantum strategies within quantum games may also inspire the development of new quantum algorithms capable of solving complex problems in polynomial time [14].

According to Sun [15] and Eisert [16], the fundamental computation process for a quantum game is given. This process involves three key steps: initial calculation of quantum strategy states, final calculation of quantum strategy states, and computation of quantum strategy representations.

In this paper, the application of quantum potential games is explored, aimed at minimizing congestion in airport common areas. The players have identical interests and they share the same utility function. It introduces a metric where passenger preferences dictate their visitation patterns to mitigate congestion. Passengers have the option to select between different common areas based on their preferences. A quantum congestion game with 10 qubits/players was implemented, where each player had access to two strategies. Leveraging the principle of quantum superposition, the game allows players to explore multiple strategies simultaneously. Simulation iterations amount to 1000, which is enough to show the behavior of the system. We used the Qiskit AER simulator for our experiments.

The research establishes that the proposed game operates as a quantum potential game designed to address congestion, ensuring the existence of a Nash equilibrium with aligned interests. Quantum entanglement is leveraged to substantiate this approach, demonstrating its efficacy through simulated results. These findings offer a robust framework for enhancing passenger flow efficiency and alleviating congestion in airport common areas using quantum game theory. Also, this paper shows the fact that the Lyapunov stability is satisfied.

The contributions of this paper are as follows:

- The system is formulated as a potential game with identical interests and a shared utility function.
- Quantum entanglement shows the efficiency of the simulated results in terms of identifying the Nash equilibrium. The flow of the passengers is optimized to reduce congestion.
- The system is Lyapunov-stable.

The remaining of this paper consists of the following: Section 2 provides the related work, Section 3 gives the background to this approach, Section 5 gives the results, Section 6 provides an analysis of the results and future directions, and Section 7 outlines the conclusions and future work.

2. Related Work

This section provides an overview of quantum approaches in wireless communications and congestion/routing in data networks. Moreover, particular transportation problems are addressed.

In [17], the authors introduce an innovative Stackelberg signaling framework to address inefficiencies in selfish routing when behavioral agents are involved. This approach models the interaction between the system and quantal response travelers as a Stackelberg game. They developed two new approximate algorithms, LoRI-v1 and LoRI-v2, which generate strategic, personalized information on the network state to guide travelers' route choices toward a socially optimal outcome. They evaluated LoRI's performance against Dijkstra's algorithm and full information disclosure on both a multi-modal Wheatstone network and a section of the Manhattan network, highlighting performance improvements and the trade-offs between system costs and runtime achieved by LoRI. Limitations of this work encompass the computational efficiency and scalability, with many travelers. Also, different levels of rationality are necessary, along with intervention types, like routing recommendations and network state information.

In [18], the authors introduce a secure vehicular quantum communication protocol designed to prevent chain-reaction collisions on foggy highways. This protocol is resistant to quantum computer attacks and uses the Nash equilibrium to counteract collusion attacks. Within the protocol, each vehicle in a cluster collaborates to compute the secret signal with minimal computational and storage requirements. This approach is particularly suited for high-traffic, foggy highways to help prevent chain collisions. Notably, the protocol does not rely on vehicle-to-vehicle or vehicle-to-infrastructure authentication, which may affect the complexity and the quality of the problem solution.

In [19], the authors provide a brief survey on the use of quantum computing in intelligent transportation systems. In particular, as it can be seen in the references therein, the authors give applications on traffic management methods, including the quantum genetic algorithm-learning vector, quantum-behaved particle swarm optimization (PSO) and radial basis function neural networks. Certainly, the practical implementation of these works need handling, and not just as solutions. Other applications include vehicular routing using the multi-objective quantum evolutionary algorithm and quantum PSO. Limitations of these works may include their performance in practical scenarios due to the complexity of the quantum part of the solutions. Finally, the authors address problems related to autonomous driving, providing insights on several critical areas, such as perception and sensor fusion, path planning, and cybersecurity.

In [20], the authors examine how quantum computing could transform logistics and supply chain management, with a focus on its potential for solving complex optimization challenges. It introduces fundamental quantum computing concepts, particularly quantum annealing and gate-based quantum computing, and highlights key algorithmic approaches, including the quantum approximate optimization algorithm and quantum annealing. The paper reviews quantum methods for addressing routing, logistics network design, fleet maintenance, cargo loading, prediction, and scheduling. Notably, most existing solutions are hybrid, leveraging both quantum and classical computing. The conclusion underscores quantum computing's early development stage. However, despite its potential, quantum computing still faces hardware limitations, requiring further advancements for practical applications.

Kocher et al. [21] emphasize the crucial role of relay selection in wireless networks and the importance of the Nash equilibrium in optimizing network performance. Their research demonstrates a quantum-accelerated PSNE (Pure Strategy Nash Equilibrium) search. By leveraging quantum computing, wireless networks can choose from a larger pool of relays with reduced latency. This game model ensures balanced traffic distribution across available relay nodes and maximizes overall utility. This research may need to make a comparison of the runtime and cost of game simulations, which will offer valuable insights into the optimal scenarios for utilizing a quantum computer for Pure Strategy Nash Equilibrium (PSNE) searches. It would also be interesting to investigate the Quantum Game Algorithm, extended and applied to games in higher dimensions.

Silva et al. [22] propose a novel framework based on a quantum game model, where network packets compete selfishly for the fastest route. Simulations indicate that the quantum version significantly outperforms the classical game model in terms of final network routing and travel times, despite both models having the same packet transmission options. We analyze Pareto optimality and the Nash equilibrium, as well as the impact of simulated and real noise on the quantum protocol. This research opens the door to developing full-stack protocols that leverage quantum properties to optimize communication systems. Due to its versatility, this game theory approach can enhance performance in both classical complex networks and future quantum networks, thereby maximizing the potential of the quantum internet. Limitations of this world emerged from the hardware restrictions on quantum computing applications and how to address them in low-cost hardware components.

Agustin et al. [23] propose a novel framework to address congestion in communication networks using a quantum game theory model, where network packets selfishly compete for the shortest route. Simulations demonstrate that the quantum game model significantly outperforms classical approaches in terms of final network routing and travel times. This opens up opportunities to develop full-stack protocols that leverage quantum properties for optimizing communication systems. Due to its versatility, this game-theoretic approach can be applied to both local networks and more complex networks, such as 5 G or LTE-A. It would be interesting in this work to determine the Nash equilibrium and assess whether it also aligns with a Pareto optimum. Another aspect would be to calculate the shortest path using quantum algorithms, fully leveraging the advantages of quantum computing to develop effective routing protocols.

3. Background to the Approach

3.1. Congestion Games

The hallmark feature of a congestion game lies in representing players' strategies Ss through a set of resources or elements R. Each resource $r \in R$ is linked with a congestion function $c_r : \mathbb{N} \to (R)$, where $c_r(k)$ signifies the congestion on resource r with $k \geq 0$ players. The player's permissible actions are delineated as various subsets of $\mathbb{R}$, constituting the agent's action set $A_i \subseteq 2^R$. Finally, the player's individual cost function, given a joint action $s = (s_i, s_{-i})$, is expressed as

$$J_i(s_i, s_{-i}) = \sum_{r \in s_i} c_r(|s|_r), \tag{1}$$

where $|s_i|_r := |j \in N : r \in s_j|$ is the player's cost on a particular resource. It is determined by the number of agents selecting that resource. Thus, in a congestion game, a player's cost on a resource depends solely on two factors, namely the identity of the resource itself and the quantity of other players selecting that resource, disregarding the identities of those players.

To calculate a player's overall individual cost, we sum the costs of the resources they have selected. When analyzing congestion games, a common inquiry revolves around assessing the "efficiency" of a specific action profile concerning the following system-level objective function, given by

$$C(s) := -\sum_{i \in N} J_i(s) = \sum_{r \in R} |a|_r \dot{c}_r(|s|_r) \tag{2}$$

3.2. Identical Interest Games (IIG)

In a class of games called "identical interest games," the existence of Pure Nash Equilibria (PNEs) is guaranteed. In these games, all players share a unified utility function. Denoting the player set as $N = \{1, ..., n\}$ and the player action sets as A_i, we express this shared utility function as $u : A \to \mathbb{R}$. This utility function applies to each player, ensuring that for any $i, j \in N$ and action profile $s \in A$, the following condition holds:

$$u_i(s) = u_j(s) = u(s) \tag{3}$$

We denote a utility maximization strategy profile as $s^* \in \arg\max_{s \in A} u(s)$; this profile must satisfy the following for every i and every s_i', as follows:

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i^{'}, , s_{-i}^*) \tag{4}$$

In every identical interest game, there exists at least one Pure Nash Equilibrium (PNE). Specifically, the action profile that maximizes the common utility function.

3.3. Quantum Potential Games

Consider a quantum potential game with N players. Let A_i be finite-dimensional quantum register associated with player i, where $i = 1, 2, ..., N$. Suppose we have a potential function $V : D(A_1) \times D(A_2) \times ... \times D(A_N) \to \mathbb{R}, (\rho_1, \rho_2, \rho_N \to Tr(R(\rho_1 \otimes \rho_2, ... \otimes \rho_N))$ for some Hermitian operator R, where $D(A_i)$ represents the space of density operators (states) on register A_i. For each player $i \in 1, 2, ..., N$, let $S_i = D(A_i)$ be the strategy set.

The utility function for player i, denoted as $u_i : S_1 S_2 ... S_N \to \mathbb{R}$, quantifies the payoff or utility player i receives based on the joint strategies of all players. Each player's utility function depends on their own strategy and the strategies chosen by the other players.

A quantum potential game with potential V for N players satisfies the following condition: For any strategy profile $s = (s_1, s_2, ..., s_N)$ and any alternative strategy profile

$s' = (s'_1, s'_2, ..., s'_N)$ that differs only in player i's strategy, the change in player i's utility is equal to the change in the potential function:

$$u_i(s, s^(-i)) - u_i(s', s^(-i)) = V(s, s^(-i)) - V(s', s^(-i)) \tag{5}$$

where $s^(-i)$ represents the strategy profile of all players except player i, for all players, $s_i \in S_{-i}$ and $s, s^{'} \in S_i$.

Quantum potential games belong to the broader category of potential games and thus share an equivalent feature known as coordination–dummy separability [24]; the utility of each player can be decomposed into a coordination component (uniform across all players and represented by the potential (V) and a dummy component (solely influenced by the actions of other players). In other words, each player's utility can be expressed as the sum of the following two distinct terms;

$$u_i(s) = V(s) + D_i(s_{-i}) \tag{6}$$

Due to coordination–dummy separation, the gradients of player i's utility function $u_i(s)$ and the potential function $V(s)$ with respect to their individual strategies are identical. This means that the trajectories followed by players' strategies under first-order learning dynamics will remain consistent, regardless of whether they participate in the potential game or the IIG where each player's utility corresponds to V. Essentially, here, the Nash equilibria of the game are the strategy profiles that correspond to the players' strategies being the best responses in comparison to the others [25].

4. Quantum Congestion Games for Passenger Routing

A cost metric was devised for routing passengers within airport areas that include stores. This algorithm guides passengers according to their preferences and takes into account various factors such as the current occupancy of each store, the presence of waiting passengers outside the stores, and the number of passengers en route to each store. Notably, the occupancy and queue are obtained from camera sensors, while the incoming passengers are coming from an Android application by setting their preferences. The formula for calculating the cost metric for each store is given in (7) [26,27].

$$F_i = \begin{cases} (P_{i,w} + P_{i,g} + \frac{P_{i,in}}{P_{i,max}}) * T_i, P_{i,max} = P_{i,in} \\ (P_{i,g} + \frac{P_{i,in}}{P_{i,max}}) * T_i - \alpha, P_{i,max} > P_{i,in} \end{cases} \tag{7}$$

where

$P_{i,w}$ is the number of passengers waiting to enter the store i;
$P_{i,g}$ is the number of passengers on route to the store i;
$P_{i,in}$ is the number of passengers inside the store i;
$P_{i,max}$ is the maximum allowed capacity of the store i;
T_i is the time that one passenger spends in the store i;
α is a negative constant (e.g., $-10,000$).

The constant α is participate in the formula in order to route the passenger to stores that are not full (maximum capacity limit). Note that the strategy of each passenger is to select the store on their mobile phone. The strategy is whether the passenger will go to common place A or whether they will go to common place B (note A).

Here, we assume that we have N players that create their routes in an airport in order to visit common places without being congested. This means that if we consider fixed routes from each common place to another as edges and the places as vertices, each edge must not be overloaded, resulting in congestion in a vertex. Initially, for the congestion game to guarantee a pure Nash equilibrium, we need to show that it is a potential game.

Theorem 1. *Consider a congestion game as described in the previous section. This congestion game is a quantum potential game with the following potential function:*

$$V(s) = \sum_{r \in R} \sum_{k=1}^{|s|_r} c_r(k), \tag{8}$$

which is essentially the common utility function given by

$$u(S) = -V(S) \tag{9}$$

Proof. We make the following assumptions to prove the theorem. The number of Ns is even, and $|\psi\rangle$ is symmetric. If N is odd and the state is not symmetric, then the quantum state $|\psi\rangle$ after a player changes their strategy from 0 to 1, or vice versa, will be a superposition of states where an odd number of qubits are in the state $|\psi\rangle$. In the case we are studying, when computing the shared utility, we need to show that the congestion game is a potential game with quantum strategies. We have an initial shared state, which is given by

$$|\psi\rangle = \tfrac{1}{\sqrt{2}}(|00...0\rangle + |11...1\rangle) \tag{10}$$

Each player can apply the unitary operations coming from these strategies, i.e., $u_i \in (I, X)$. The initial joined state is given by Equation (10). The initial shared utility is given by $\langle\psi|\hat{H}|\psi\rangle$.

When player i changes their strategy from I to X, we have a new state that is given by

$$|\psi'\rangle = (X_i \otimes I \otimes ... \otimes I)|\psi\rangle = \frac{1}{\sqrt{2}}(|1_i...0\rangle + |0_i...1\rangle). \tag{11}$$

Thus the new shared utility after some calculations is 1, and the change in the shared utility function is 0. This is due to the fact that the number of qubits is odd. In the state $|\rangle$, the change in the shared utility is given by

$$u = \langle\psi'|\hat{H}|\psi'\rangle = \frac{1}{2} + \frac{1}{2}. \tag{12}$$

The potential function that corresponds to the shared utility is given by

$$V(\psi) = \langle\psi|\hat{H}|\psi\rangle. \tag{13}$$

If we take $V = u = H$, the change in the utility function is given by

$$\Delta V = \langle\psi'|\hat{V}|\psi'\rangle - \langle\psi\hat{V}|\psi\rangle = u' - u. \tag{14}$$

The fact that the change in the utility function is the same as with the change in the potential function shows that the game is an exact potential game. □

Also, we have the following property of the system that is under investigation:

Theorem 2. *The system is Lyapunov-stable.*

Proof. Non-negativity of the potential function: The potential function is given by

$$V(\psi) = \langle\psi|\hat{H}|\psi\rangle. \tag{15}$$

Assuming that $\hat{H}$ represents costs or energies which are non-negative, $V(\psi) \geq 0$ for all states $|\psi\rangle$.

At equilibrium, the potential function $V(\psi)$ is minimized. The equilibrium condition is

$$V(\psi) = \min_{|\psi'\rangle} V(\psi'). \tag{16}$$

This implies that the utility function $u = -V(\psi)$ is maximized, indicating that no player has an incentive to change their strategy.

Equilibrium condition:: At equilibrium, the potential function $V(\psi)$ is minimized. The equilibrium condition is

$$V(\psi) = \min_{|\psi'\rangle} V(\psi'). \tag{17}$$

This implies that the utility function $u = -V(\psi)$ is maximized, indicating that no player has an incentive to change their strategy.

Decreasing behavior of the potential function: When a player changes their strategy, the new state $|\psi'\rangle$ is

$$|\psi'\rangle = (X_i \otimes I \otimes \cdots \otimes I)|\psi\rangle. \tag{18}$$

The change in the potential function is

$$\Delta V = V(\psi') - V(\psi). \tag{19}$$

Since $u = -V(\psi)$, the change in utility is

$$\Delta u = u' - u = -\Delta V. \tag{20}$$

Therefore, if $\Delta u \geq 0$, then $\Delta V \leq 0$. The potential function decreases or remains constant over time, ensuring that the system moves towards lower potential states.

Thus, the system is Lyapunov-stable because the potential function $V(\psi)$ is non-negative, reaches a minimum at equilibrium, and decreases over time as strategies are adjusted. □

5. Results

We set up a quantum congestion game with 10 qubits/players and two strategies available per player. The quantum congestion game utilizes the principle of quantum superposition to enable players to explore multiple strategies simultaneously. We used the Qiskit AER simulator for our experiments.

By running simulations with 1000 iterations and making iterative adjustments, players seek to reach an equilibrium where the potential function stabilizes. This stabilization indicates an optimal distribution of resources with minimal congestion. The final strategies plot illustrates the stable state of the game, showcasing the collective behavior of players in response to shared utility dynamics.

A quantum circuit was created where each of the 10 qubits (players) was initialized in a superposition state using a Hadamard gate. This allowed the players to explore both strategies (0 and 1) simultaneously. The quantum circuit was simulated using a quantum simulator with 1000 shots to measure the outcomes. The measurement results provided the frequency of each possible strategy combination across all players.

The results from the quantum circuit simulation were analyzed to determine the frequency of each strategy combination. Each strategy was represented as a binary string of length 10, where each bit corresponded to the strategy chosen by a player (0 or 1).

The frequencies were used to calculate the utility for each strategy combination. The utility for each strategy was calculated as the proportion of players choosing strategy 0 and strategy 1, normalized by the total number of players.

In Figure 1, the reader can see the value of the potential function over the iterations. The fact that the potential function seems to fluctuate may be attributed to specific reasons.

The potential function may not be smooth enough to show convergence clearly. Smoothing the potential function is one reason whereby using a moving average or smoothing technique to visualize the trend in the potential function could be useful. Moreover, more gradual updates, instead of binary updates (0 or 1), can allow fractional updates that gradually steer the strategies toward a convergent state. In Figure 2, the potential function is smoothed and the strategy changes are based on utility differences rather than random flipping, making the plot appear smoother.

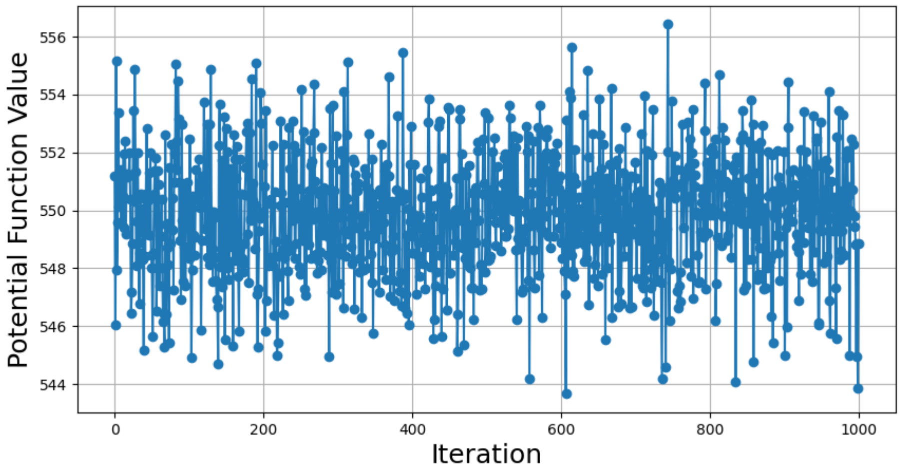

Figure 1. Value of the potential function: non-smooth.

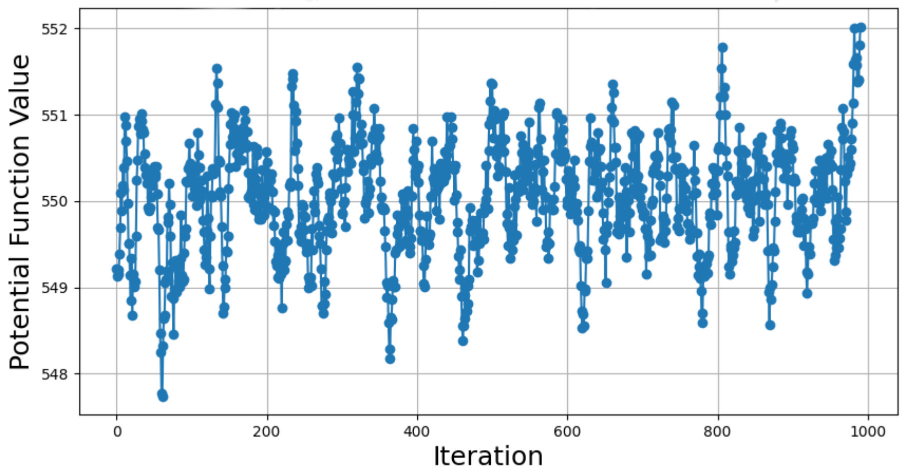

Figure 2. Value of the potential function: smooth.

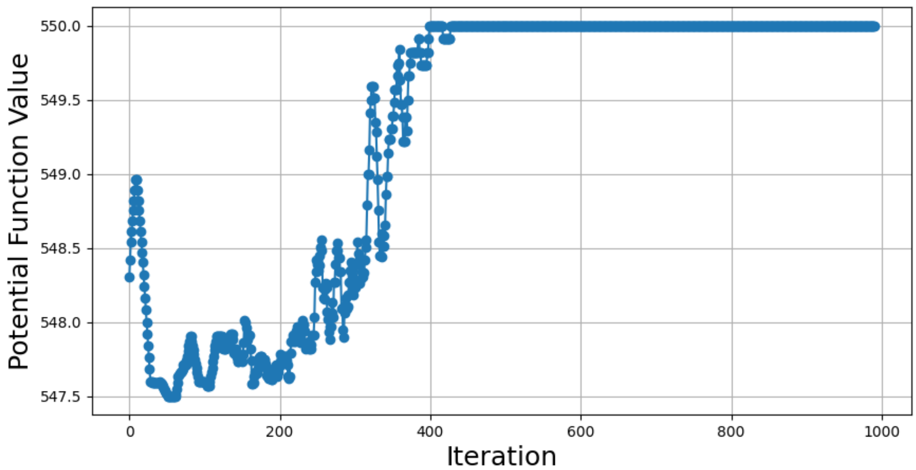

Figure 3. Value of the potential function: simulated annealing.

Also, we experimented with the SA method to see the difference with a well-established method. Simulated annealing is used for optimization by gradually lowering a temperature parameter, which initially allows the system to explore various states, even those that do not immediately improve utility. In the early stages, a high temperature enables random shifts in strategies, encouraging broad exploration. As temperature decreases, the system becomes more selective, preferring only utility-enhancing changes, which steers it toward stability and high utility. This cooling schedule drives convergence, narrowing state changes to those that maximize or stabilize the potential function. As a result, the potential function smoothly ascends toward a peak, achieving an optimal and stable configuration with minimal fluctuations.

In comparison with the quantum game, it uses a quantum circuit that places passengers in superpositions, blending multiple strategies rather than fixing one choice. Upon measurement, these superpositions collapse into specific outcomes with probabilistic results, adding inherent randomness to each iteration. This probabilistic measurement causes fluctuations in strategy distributions, leading the potential function to oscillate rather than steadily increase or converge, as it was observed in Figure 3. Each iteration's strategy is updated based on superposition evaluations and utility recalculations and introduces variability, producing a potential function that fluctuates, reflecting the high variability from quantum sampling.

In Table 1, the reader can observe that final strategies are followed by each player with a random flip, which is utility-based and in accordance with the SA. It is clear that the SA method assigns every passenger to visit the same common place even though this is not what is necessary for this particular case. The random flip shows better allocations since it enforces some of the passengers not to visit the same common place. Finally, the quantum game exhibits better behavior by allocating passengers to another common place in higher number, distributing the passengers more evenly to common places.

Finally, in Figure 4, the reader can see the evolution of the strategies followed in the first experiment. There is a fluctuation in the strategies ranging from 0 to 1, as shown by the fluctuations in the potential function as well. Note that the color of the plot show the intention to follow 0 (dark green) when the color is darker and the intention to follow 1 when the color is lighted (light green). This shows the dynamic nature of the system since the place will become overcrowded if all the passengers select to move to a common place all the time.

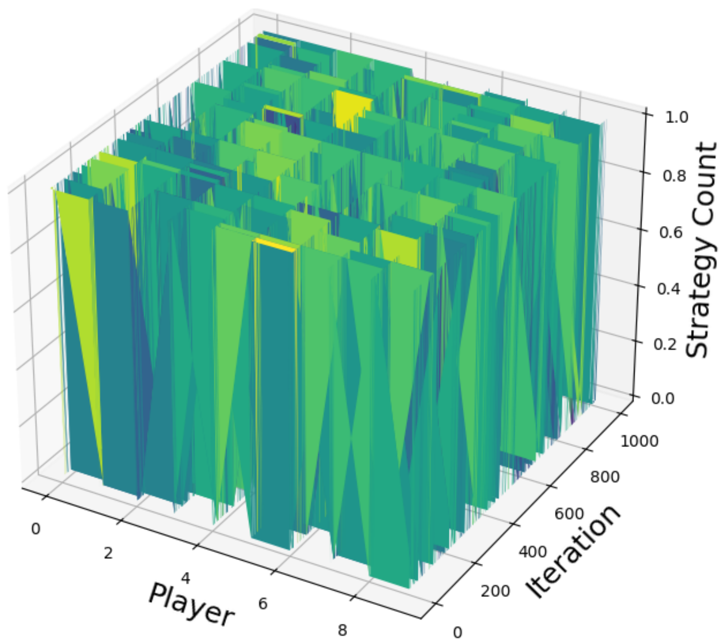

Figure 4. Evolution of strategies: utility based.

In Figure 5, the reader can see the evolution of strategies when the SA method is utilized. As indicated in the potential function values, there is an initial fluctuation in the strategies, and then the system converges to a stable state. Here, the remark is that all the passengers select to visit the same common place, which may not be the best case to minimize overcrowding. This aligns with the aforementioned table values, which showed no difference in the final selection after the end of the iterations.

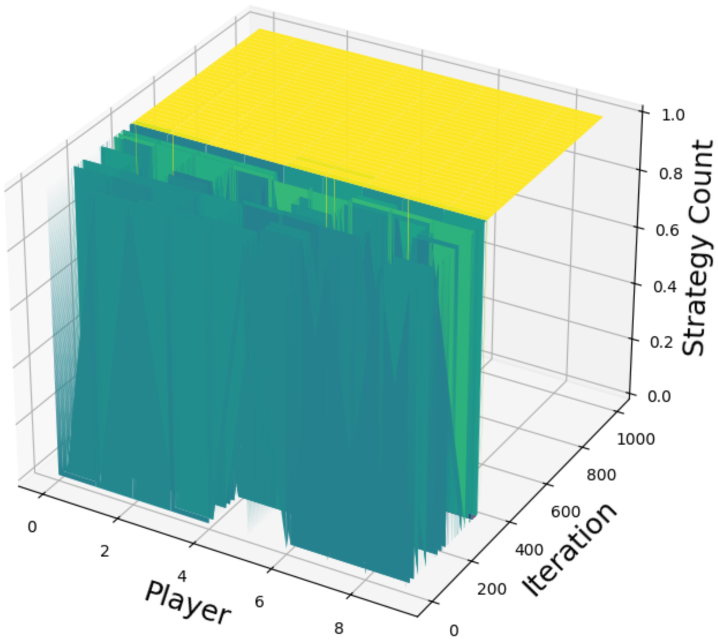

Figure 5. Evolution of strategies: SA.

Table 1. Final strategies with the random flip, utility-based and SA.

Passenger	Random Flip (Strategy)	Utility-Based (Strategy)	SA-Based (Strategy)
1	1	1	1
2	1	1	1
3	1	1	1
4	0	1	1
5	1	1	1
6	0	0	1
7	0	1	1
8	1	0	1
9	1	0	1
10	1	0	1

6. Analysis of Results and Future Research

The setup of the experiment of this paper involved initializing each qubit in a superposition, where players could simultaneously consider strategies “0” and “1”. Using 1000 shots, we measured outcomes to determine the frequency of strategy combinations for each player, representing them as binary strings. These frequencies were then used to calculate each strategy’s utility, normalized by the total number of players selecting each option.

The resulting potential function values over the iterations showed notable fluctuations, attributed to the inherent variability in quantum sampling. A smoothing technique was then applied to the potential function, yielding a clearer convergence trend, as utility-based updates provided more consistent strategy adjustments compared to random flipping.

Additionally, we experimented with the SA method as a benchmark. SA's cooling schedule initially allowed random strategy shifts, encouraging broad exploration. As the temperature lowered, the system selectively favored utility-enhancing adjustments, gradually converging to a stable, optimal configuration with minimal fluctuations. Unlike the quantum game, which retained probabilistic variations due to the superposition, the SA method showed steady convergence without significant oscillations, indicating a more straightforward path to stability.

The quantum game displayed a dynamic distribution across strategies, resulting in a more balanced allocation that could prevent the overcrowding of common spaces. In contrast, the SA method guided all players to the same outcome, potentially increasing congestion in a single location, underscoring the quantum game's potential advantage in managing distributed player behavior effectively.

For a future work, our aim is to enhance this work by providing the price and anarchy and the price of stability. Moreover, learning will be also introduced to investigate the dynamics of the quantum potential game. Non-binary strategies, as given in Equation (7), will be trellised to investigate the quantum states that will be created and check the complexity of the algorithm. Finally, a practical implementation will be attempted to investigate potential bottlenecks of the approach.

In terms of the current state of the art in the research of quantum games, the related work emphasizes of both quantum works and quantum games. To our knowledge, minimizing crowds has not been addressed using quantum game theory to avoid airborne disease spreading. Hence, the related work is mainly on quantum games utilized in other networks. Identical interest games may be an enhancement since they add simplicity to the problem and involve binary strategies, with the existing works suffering from limitations to hardware utilization and complexity. Here, we assume that the computers utilized in the scenario are powerful since they can be embedded into the airport's infrastructure. Moreover, congestion games in classical game theory are well known for consisting of a Nash equilibrium. Hence, the whole proposal was formulated to simplify the problem of quantum circuits being applied to a problem. Algorithms, such as PSO or genetic algorithms, will be employed towards solving this problem in the future, since they offer solutions that are not limited to local minima.

Moreover, quantum potential games differ from classical quantum game models by offering greater adaptability and computational efficiency in real-time environments, like airports. While classical quantum games often focus on static equilibrium solutions, quantum potential games leverage dynamic interactions, allowing for continuous adaptations to occur regarding congestion and passenger behavior variations without extensive interventions. This approach supports high scalability and flexible optimization for passenger flow and layout adjustments, unlike classical models, which may lack responsiveness to fluctuating real-time conditions.

7. Conclusions

In this paper, quantum potential games were addressed for minimizing congestion within common areas in airports. A quantum congestion game with 10 qubits/players was implemented, with each player having access to two strategies. By leveraging quantum superposition, players can explore multiple strategies simultaneously. The simulation runs for 1000 iterations, providing sufficient insights into the system's behavior.

By setting up preferences, we showed a metric that encompassed the visits of a passenger to a common place. A passenger can set that they will visit a common place or go to another common place. This study shows that the game suggested is a potential game, whereby a Nash equilibrium exists. The quantum entanglement provides the proof of concept, and results are given for the specific approach used. We used the SA method to show a comparison with the approach followed.

Due to the fluctuations of the players' strategies between 0 and 1, the evolution of the potential function value fluctuated as well, while the final strategies, after 1000 iterations, showed that some passengers visited place A while others did not. In general, the proposed system is structured as a potential game characterized by aligned interests and a shared utility function, promoting cooperative outcomes. Quantum entanglement enhances the efficiency of the simulation, particularly in identifying the Nash equilibrium, which optimizes passenger flow and mitigates congestion. Additionally, the system maintains Lyapunov stability, ensuring its resilience and stability under operational dynamics.

Limitations of the proposed approach can be summarized as follows: Simulating quantum circuits with many qubits is highly computationally intensive and requires specialized quantum hardware for scalability, as classical simulations quickly become impractical. Furthermore, quantum games introduce probabilistic outcomes due to superposition and measurement, leading to fluctuating strategies and potential values that make stable convergence difficult, especially when utility fluctuations are high. Moreover, while potential games are simpler with binary or limited strategies, adding more strategies increases the complexity of utility evaluations and strategy updates, making practical implementations challenging. In multi-strategy scenarios, quantum circuits become especially hard to scale due to the exponential growth that occurs in quantum states.

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